

GROUPS OF ORDERS 33-47

SUMMARY

order	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
# abelian	1	1	1	4	1	1	1	3	1	1	1	2	2	1	1
# other decomposable	0	0	0	6	0	0	0	6	0	3	0	1	0	0	0
# other indecomp	0	1	0	5	0	1	1	5	0	2	0	1	0	1	0
TOTAL	1	2	1	15	1	2	2	14	1	6	1	4	2	2	1

$$33 = \langle A^{33} \rangle$$

$$34 = \langle A^{34} \rangle$$

$$34.2 = D_{34} = \langle A^{17}, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10
elts	1	A	A ²	A ³	A ⁴	A ⁵	A ⁶	A ⁷	A ⁸	A ⁿ B
		A ¹⁶	A ¹⁵	A ¹⁴	A ¹³	A ¹²	A ¹¹	A ¹⁰	A ⁹	

C	1	2	3	4	5	6	7	8	9	10	
#	1	2	2	2	2	2	2	2	2	17	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	-1	1
χ_3	2	c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	c ₇	c ₈	0	0
χ_4	2	c ₂	c ₄	c ₆	c ₈	c ₇	c ₅	c ₃	c ₁	0	0
χ_5	2	c ₃	c ₆	c ₉	c ₅	c ₂	c ₁	c ₄	c ₇	0	0
χ_6	2	c ₄	c ₈	c ₅	c ₁	c ₃	c ₇	c ₆	c ₂	0	0
χ_7	2	c ₅	c ₇	c ₂	c ₃	c ₈	c ₄	c ₁	c ₆	0	0
χ_8	2	c ₆	c ₅	c ₁	c ₇	c ₄	c ₂	c ₈	c ₃	0	0
χ_9	2	c ₇	c ₃	c ₄	c ₆	c ₁	c ₈	c ₂	c ₅	0	0
χ_{10}	2	c ₈	c ₁	c ₇	c ₂	c ₆	c ₃	c ₅	c ₄	0	0
^	1	17	17	17	17	17	17	17	17	2	

$$c_k = 2\cos(2k\pi/17)$$

	Classes	H	G/H	\cap
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	17	2	1 G'

Sylow and maximal subgroups: $[2] \times 17$, $[17]$

$$35 = \langle A^{35} \rangle$$

$$36 = \langle A^{36} \rangle = 9 \times 4$$

$$36.02 = 12 \times 3 = \langle A^{12}, B^3 \rangle$$

$$36.03 = 18 \times 2 = \langle A^{18}, B^2 \rangle$$

$$36.04 = 6^2 = \langle A^6, B^6 \rangle$$

$$36.05 = D_6 \times 6 = \langle A^3, B^2, C^3, D^2, BA = A^{-1}B \rangle$$

$$36.06 = D_{3,4} \times 3 = \langle A^3, B^4, C^3, BA = A^{-1}B \rangle$$

$$36.07 = A_4 \times 3 = \langle A^2, B^2, C^3, D^3, CA = ABC, CB = AC \rangle$$



$$36.08 = \langle A^9, B^2, C^2, BA = AC, CA = ABC \rangle$$

	1	2	3	4	5	6	7	8	9	10	11	12
1	A^3	A^6	B	A^3B	A^6B	AB^mC^n	$A^4B^mC^n$	$A^7B^mC^n$	$A^2B^mC^n$	$A^5B^mC^n$	$A^8B^mC^n$	
			C	A^3C	A^6C							
			BC	A^3BC	A^6BC							

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	1	1	3	3	3	4	4	4	4	4	4	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	ω	ω	ω	ω^2	ω^2	ω^2	1
χ_3	1	1	1	1	1	1	ω^2	ω^2	ω^2	ω	ω	ω	1
χ_4	1	ω	ω^2	1	ω	ω^2	θ	θ^4	θ^7	θ^2	θ^5	θ^8	2
χ_5	1	ω	ω^2	1	ω	ω^2	θ^4	θ^7	θ	θ^8	θ^2	θ^5	2
χ_6	1	ω	ω^2	1	ω	ω^2	θ^7	θ	θ^4	θ^5	θ^8	θ^2	2
χ_7	1	ω^2	ω	1	ω^2	ω	θ^2	θ^8	θ^5	θ^4	θ	θ^7	2
χ_8	1	ω^2	ω	1	ω^2	ω	θ^5	θ^2	θ^8	θ	θ^7	θ^4	2
χ_9	1	ω^2	ω	1	ω^2	ω	θ^8	θ^5	θ^2	θ^7	θ^4	θ	2
χ_{10}	3	3	3	-1	-1	-1	0	0	0	0	0	0	3
χ_{11}	3	3ω	$3\omega^2$	-1	$-\omega$	$-\omega^2$	0	0	0	0	0	0	0
χ_{12}	3	$3\omega^2$	3ω	-1	$-\omega^2$	$-\omega$	0	0	0	0	0	0	0
^	1	3	3	2	6	6	9	9	9	9	9	9	

$$\theta = e^{2\pi i/9}$$

	Classes	H	G/H	\cap
1	1 + 2 + 3 + 4 + 5 + 6	6×2	3	1
2	1 + 4	2^2	9	2 G'
3	1 + 2 + 3	3	A_4	3 $Z = \Phi = \mathfrak{K}$

Sylow subgroups: $[2^2]$, $[9] \times 4$ Maximal subgroups: $[6 \times 2]$, $[D_6] \times 4$

$$36.09 = 18.4 \times 2 = \langle A^3, B^3, C^2, D^2, CA = A^{-1}C, CB = B^{-1}C \rangle$$

$$36.10 = \langle A^3, B^3, C^4, CA = A^{-1}C, CB = B^{-1}C \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11	12
#	1	C ²	A	AC ²	B	BC ²	AB	ABC ²	AB ²	AB ² C ²	A ^m B ⁿ C	A ^m B ⁿ C ³
		A ³	A ² C ²	B ²	B ² C ²	A ² B ²	A ² B ² C ²	A ² B	A ² BC ²			

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	1	2	2	2	2	2	2	2	2	9	9	ℳ
χ ₁	1	1	1	1	1	1	1	1	1	1	1	1	G
χ ₂	1	1	1	1	1	1	1	1	1	1	-1	-1	1
χ ₃	1	-1	1	-1	1	-1	1	-1	1	-1	i	-i	2
χ ₄	1	-1	1	-1	1	-1	1	-1	1	-1	-i	i	2
χ ₅	2	2	2	2	-1	-1	-1	-1	-1	-1	0	0	3
χ ₆	2	2	-1	-1	2	2	-1	-1	-1	-1	0	0	4
χ ₇	2	2	-1	-1	-1	-1	2	2	-1	-1	0	0	5
χ ₈	2	2	-1	-1	-1	-1	-1	-1	2	2	0	0	6
χ ₉	2	-2	2	-2	-1	1	-1	1	-1	1	0	0	7
χ ₁₀	2	-2	-1	1	2	-2	-1	1	-1	1	0	0	8
χ ₁₁	2	-2	-1	1	-1	1	2	-2	-1	1	0	0	9
χ ₁₂	2	-2	-1	1	-1	1	-1	1	2	-2	0	0	10
^	1	2	3	6	3	6	3	6	3	6	4	4	

	Classes	H	G/H	K
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	18.18	2	1
2	1 + 3 + 5 + 7 + 9	3²	4	2
3	1 + 2 + 3 + 4	6	D₆	3
4	1 + 2 + 5 + 6	6	D₆	4
5	1 + 2 + 7 + 8	6	D₆	5
6	1 + 2 + 9 + 10	6	D₆	6
7	1 + 3	3	D_{3,4}	7
8	1 + 5	3	D_{3,4}	8
9	1 + 7	2	18.4	9
10	1 + 9	3	12.4	10
11	1 + 2	3	D_{3,4}	3 ∩ 4 Z = Φ = ℳ

Sylow subgroups: [4] × 9, [3²] Maximal subgroups: [6 × 3], [D_{3,4}] × 2

$$36.11 = D_{36} = D_{18} \times 2 = \langle A^9, B^2, C^2, BA = A^{-1}B \rangle$$

$$36.12 = D_{9,4} = \langle A^9, B^4, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11	12
elts	1	B ²	A ³ A ⁶	A ³ B ² A ⁶ B ²	A A ⁸	AB ² A ⁸ B ²	A ² A ⁷	A ² B ² A ⁷ B ²	A ⁴ A ⁵	A ⁴ B ² A ⁵ B ²	A ⁿ B	A ⁿ B ³

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	1	2	2	2	2	2	2	2	2	9	9	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	-1	-1	1
χ_3	1	-1	1	-1	1	-1	1	-1	1	-1	i	-i	2
χ_4	1	-1	1	-1	1	-1	1	-1	1	-1	-i	i	2
χ_5	2	2	2	2	-1	-1	-1	-1	-1	-1	0	0	3
χ_6	2	-2	2	-2	-1	1	-1	1	-1	1	0	0	4
χ_7	2	2	-1	-1	c ₁	c ₁	c ₂	c ₂	c ₄	c ₄	0	0	5
χ_8	2	2	-1	-1	c ₄	c ₄	c ₁	c ₁	c ₂	c ₂	0	0	5
χ_9	2	2	-1	-1	c ₂	c ₂	c ₄	c ₄	c ₁	c ₁	0	0	5
χ_{10}	2	-2	-1	1	c ₁	-c ₁	c ₂	-c ₂	c ₄	-c ₄	0	0	0
χ_{11}	2	-2	-1	1	c ₄	-c ₄	c ₁	-c ₁	c ₂	-c ₂	0	0	0
χ_{12}	2	-2	-1	1	c ₂	-c ₂	c ₄	-c ₄	c ₁	-c ₁	0	0	0
\wedge	1	2	3	6	9	18	9	18	9	18	4	4	

$$c_k = 2\cos(2k\pi/9)$$

Normal subgroups

	Classes	H	G/H	\cap
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	18	2	1
2	1 + 3 + 5 + 7 + 9	9	2 ²	2 G'
3	1 + 2 + 3 + 4	6	D ₆	3 Φ
4	1 + 3	3	D _{3,4}	4
5	1 + 2	2	D ₁₈	5 $Z = \mathcal{Z}$

Sylow subgroups: [4] \times 9, [9] Maximal subgroups: [18], [D_{3,4}] \times 3

$$36.13 = \langle A^3, B^2, C^3, D^2, BA = A^{-1}B, DC = C^{-1}D \rangle$$

	CC	1	2	3	4	5	6	7	8	9	
elts		1	A A ²	B A ² B	AB C ²	C AC ²	A ² C A ² C ²	A ⁿ BC A ⁿ BC ²	D C ² D	CD AC ⁿ D A ² C ⁿ D	A ^m BC ⁿ D

C	1	2	3	4	5	6	7	8	9	
#	1	2	3	2	4	6	3	6	9	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	-1	1	1	-1	1	1	-1	1
χ_3	2	-1	0	2	-1	0	2	-1	0	2
χ_4	1	1	1	1	1	1	-1	-1	-1	3
χ_5	1	1	-1	1	1	-1	-1	-1	1	4
χ_6	2	-1	0	2	-1	0	-2	1	0	5
χ_7	2	2	2	-1	-1	-1	0	0	0	6
χ_8	2	2	-2	-1	-1	1	0	0	0	7
χ_9	4	-2	0	-2	1	0	0	0	0	0
\wedge	1	3	2	3	3	6	2	6	2	

Normal subgroups

Classes	H	G/H	\cap
1	1 + 2 + 4 + 5 + 7 + 8	$D_6 \times 3$	2
2	1 + 4 + 7	D_6	D_6
3	1 + 2 + 3 + 4 + 5 + 6	$D_6 \times 3$	2
4	1 + 2 + 4 + 5 + 9	18.4	2
5	1 + 4	3	$D_6 \times 2$
6	1 + 2 + 3	D_6	D_6
7	1 + 2	3	$D_6 \times 2$
8	1 + 2 + 4 + 5	3×3	2^2

$1 \cap 2 \quad G'$

Sylow subgroups: $[2^2] \times 9, [3^2]$ Maximal subgroups: $[D_6 \times 3] \times 2, [18.4], [D_6 \times 2] \times 6$

$$36.14 = \langle A^3, B^3, C^4, CA = B^{-1}C, CB = AC \rangle$$

CC	1	2	3	4	5	6
elts	1	A B A ² B ²	AB AB ² A ² B A ² B ²	A ^m B ⁿ C ²	A ^m B ⁿ C	A ^m B ⁿ C ³

C	1	2	3	4	5	6	
#	1	4	4	9	9	9	\mathfrak{K}
χ^1	1	1	1	1	1	1	G
χ^2	1	1	1	1	-1	-1	1
χ^3	1	1	1	-1	i	-i	2
χ^4	1	1	1	-1	-i	i	2
χ^5	4	1	-2	0	0	0	0
χ^6	4	-2	1	0	0	0	0
^	1	3	3	2	4	4	

	Classes	H	G/H	\cap
1	1 + 2 + 3 + 4	18.4	2	1
2	1 + 2 + 3	3 × 3	4	2 G'

Sylow subgroups: $[4] \times 9, [3^2]$ Maximal subgroups: $[18.4], [4] \times 4$

$$36.15 = D_6 \times 6 = \langle A^3, B^2, C^6, BA = A^{-1}B \rangle$$

$$37 = \langle A^{37} \rangle$$

$$38 = \langle A^{38} \rangle = 19 \times 2$$



$$38.2 = D_{38} = \langle A^{19}, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11	
elts	1	A	A ²	A ³	A ⁴	A ⁵	A ⁶	A ⁷	A ⁸	A ⁹	A ¹⁰	A ¹¹
		A ¹⁸	A ¹⁷	A ¹⁶	A ¹⁵	A ¹⁴	A ¹³	A ¹²	A ¹¹	A ¹⁰	A ⁹	A ⁸

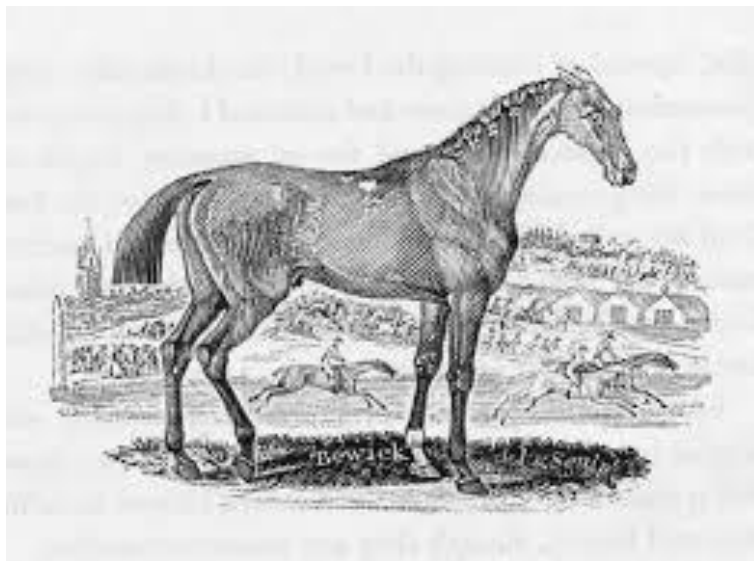
C	1	2	3	4	5	6	7	8	9	10	11	
#	1	2	2	2	2	2	2	2	2	2	19	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	-1	1
χ_3	2	c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	c ₇	c ₈	c ₉	0	0
χ_4	2	c ₂	c ₄	c ₆	c ₈	c ₉	c ₇	c ₅	c ₃	c ₁	0	0
χ_5	2	c ₃	c ₆	c ₉	c ₇	c ₄	c ₁	c ₂	c ₅	c ₈	0	0
χ_6	2	c ₄	c ₈	c ₇	c ₃	c ₁	c ₅	c ₉	c ₆	c ₂	0	0
χ_7	2	c ₅	c ₉	c ₄	c ₁	c ₆	c ₈	c ₃	c ₂	c ₇	0	0
χ_8	2	c ₆	c ₇	c ₁	c ₅	c ₈	c ₂	c ₆	c ₉	c ₃	0	0
χ_9	2	c ₇	c ₅	c ₂	c ₉	c ₃	c ₄	c ₈	c ₁	c ₆	0	0
χ_{10}	2	c ₈	c ₃	c ₅	c ₆	c ₂	c ₉	c ₁	c ₇	c ₄	0	0
χ_{11}	2	c ₉	c ₁	c ₈	c ₂	c ₇	c ₃	c ₉	c ₄	c ₅	0	0
^	1	19	19	19	19	19	19	19	19	19	2	

$$c_k = 2\cos(2k\pi/19)$$

	Classes	H	G/H	\cap
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	19	2	1 G'

Sylow and maximal subgroups: $[2] \times 19, [19]$

$$39 = \langle A^{39} \rangle = 13 \times 3$$



$$39.2 = M_{13,3}^{(3)} = \langle A^{13}, B^3, BA = A^3B \rangle$$

CC	1	2	3	4	5	6	7	
elts	1	A A ³ A ⁹	A ² A ⁵ A ⁶	A ⁴ A ¹⁰ A ¹²	A ⁷ A ⁸ A ¹¹	A ⁿ B	A ⁿ B ²	
C	1	2	3	4	5	6	7	
#	1	3	3	3	3	13	13	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	ω	ω^2	1
χ_3	1	1	1	1	1	ω^2	ω	1
χ_4	3	$\theta + \theta^3 + \theta^9$	$\theta^2 + \theta^5 + \theta^6$	$\theta^4 + \theta^{10} + \theta^{12}$	$\theta^7 + \theta^8 + \theta^{11}$	0	0	0
χ_5	3	$\theta^2 + \theta^5 + \theta^6$	$\theta^4 + \theta^{10} + \theta^{12}$	$\theta^7 + \theta^8 + \theta^{11}$	$\theta + \theta^3 + \theta^9$	0	0	0
χ_6	3	$\theta^4 + \theta^{10} + \theta^{12}$	$\theta^7 + \theta^8 + \theta^{11}$	$\theta + \theta^3 + \theta^9$	$\theta^2 + \theta^5 + \theta^6$	0	0	0
χ_7	3	$\theta^7 + \theta^8 + \theta^{11}$	$\theta + \theta^3 + \theta^9$	$\theta^2 + \theta^5 + \theta^6$	$\theta^4 + \theta^{10} + \theta^{12}$	0	0	0
\wedge	1	19	19	19	19	3	3	

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5	13	2	1	G'

Sylow and maximal subgroups: $[3] \times 13$, $[13]$

$$40 = \langle A^{40} \rangle = 8 \times 5$$

$$40.02 = 10 \times 4 = \langle A^{10}, B^4 \rangle$$

$$40.03 = 10 \times 2^2 = \langle A^{10}, B^2, C^2 \rangle$$

$$40.04 = D_8 \times 5 = \langle A^4, B^2, C^5, BA = A^{-1}B \rangle$$

$$40.05 = Q_8 \times 5 = \langle A^4, B^2 = A^2, C^5, BA = A^{-1}B \rangle$$

$$40.06 = D_{10} \times 2^2 = \langle A^5, B^2, C^2, D^2, BA = A^{-1}B \rangle$$

$$40.07 = D_{10} \times 4 = \langle A^5, B^2, C^4, BA = A^{-1}B \rangle$$

$$40.08 = D_{5,4} \times 2 = \langle A^5, B^4, C^2, BA = A^{-1}B \rangle$$

40.09 = D_{5,8} = ⟨A⁵, B⁸, BA = A⁻¹B⟩

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	B ⁴	B ²	B ⁶	A	AB ⁴	AB ²	AB ⁶	A ²	A ² B ⁴	A ² B ²	A ² B ⁶	A ⁿ B	A ⁿ B ⁵	A ⁿ B ³	A ⁿ B ⁷	
				A ⁴	A ⁴ B ⁴	A ⁴ B ²	A ⁴ B ⁶	A ³	A ³ B ⁴	A ³ B ²	A ³ B ⁶					

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		
^	1	1	1	1	2	2	2	2	2	2	2	2	2	5	5	5	5	ℳ
χ₁	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ₂	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1		1
χ₃	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	i	i	-i	-i		2
χ₄	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-i	-i	i	i		2
χ₅	1	-1	i	-i	1	-1	i	-i	1	-1	i	-i	θ	θ ⁵	θ ³	θ ⁷		3
χ₆	1	-1	i	-i	1	-1	i	-i	1	-1	i	-i	θ ⁵	θ	θ ⁷	θ ³		3
χ₇	1	-1	-i	i	1	-1	-i	i	1	-1	-i	i	θ ³	θ ⁷	θ	θ ⁵		3
χ₈	1	-1	-i	i	1	-1	-i	i	1	-1	-i	i	θ ⁷	θ ³	θ ⁵	θ		3
χ₉	2	2	2	2	c ₁	c ₁	c ₁	c ₁	-c ₂	-c ₂	-c ₂	-c ₂	0	0	0	0		4
χ₁₀	2	2	2	2	-c ₂	-c ₂	-c ₂	-c ₂	c ₁	c ₁	c ₁	c ₁	0	0	0	0		4
χ₁₁	2	2	-2	-2	c ₁	c ₁	-c ₁	-c ₁	-c ₂	-c ₂	c ₂	c ₂	0	0	0	0		5
χ₁₂	2	2	-2	-2	-c ₂	-c ₂	c ₂	c ₂	c ₁	c ₁	-c ₁	-c ₁	0	0	0	0		5
χ₁₃	2	-2	2i	-2i	-c ₂	c ₂	-ic ₂	ic ₂	c ₁	-c ₁	ic ₁	-ic ₁	0	0	0	0		0
χ₁₄	2	-2	-2i	2i	-c ₂	c ₂	ic ₂	-ic ₂	c ₁	-c ₁	-ic ₁	ic ₁	0	0	0	0		0
χ₁₅	2	-2	-2i	2i	c ₁	-c ₁	-ic ₁	-ic ₁	-c ₂	c ₂	ic ₂	-ic ₂	0	0	0	0		0
χ₁₆	2	-2	2i	-2i	c ₁	-c ₁	ic ₁	-ic ₁	-c ₂	c ₂	-ic ₂	ic ₂	0	0	0	0		0
^	1	2	4	4	5	10	20	20	5	10	20	20	8	8	8	8		

$c_k = 2\cos(2k\pi/5), \theta = e^{2\pi i/8}$

	Classes	H	G/H	∩
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12	20	2	1
2	1 + 2 + 5 + 6 + 9 + 10	10	4	2
3	1 + 5 + 9	5	8	3 G'
4	1 + 2 + 3 + 4	4	D₁₀	4 Z = Φ = ℳ
5	1 + 2	2	D_{5,4}	5

Sylow subgroups: [8] × 5, [5] **Maximal subgroups:** [20], [8] × 5

40.10 = M_{5,4}⁽³⁾ × 2 = ⟨A⁵, B⁴, C², BA = A³B⟩

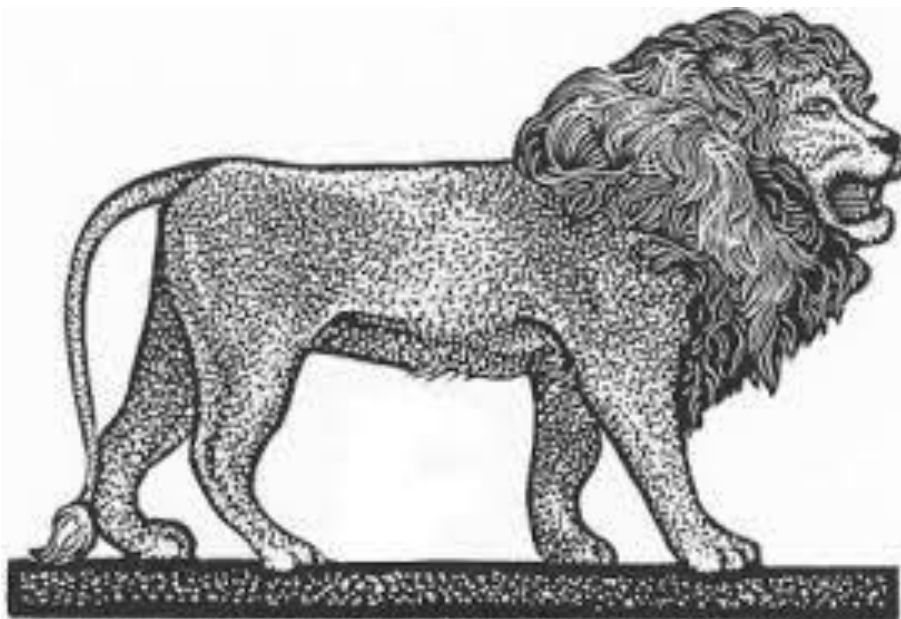
$$40.11 = M_{5,8}^{(3)} = \langle A^5, B^8, BA = A^3B \rangle$$

	1	2	3		4				5	6	7	8	9	10		
	1	B^4	A	A^2	A^3	A^4	AB^4	A^2B^4	A^3B^4	A^4B^4	A^nB^2	A^nB^6	A^nB	A^nB^5	A^nB^3	A^nB^7

C	1	2	3	4	5	6	7	8	9	10	
#	1	1	4	4	5	5	5	5	5	5	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	-1	-1	-1	-1	1
χ_3	1	1	1	1	-1	-1	i	-i	-i	i	2
χ_4	1	1	1	1	-1	-1	-i	i	i	-i	2
χ_5	1	-1	1	-1	i	-i	θ	θ^5	θ^3	θ^7	3
χ_6	1	-1	1	-1	i	-i	θ^5	θ	θ^7	θ^3	3
χ_7	1	-1	1	-1	-i	i	θ^2	θ^2	θ^2	θ^2	3
χ_8	1	-1	1	-1	-i	i	θ^2	θ^2	θ^2	θ^2	3
χ_9	4	4	-1	-1	0	0	0	0	0	0	4
χ_{10}	4	-4	-1	-1	0	0	0	0	0	0	0
^	1	2	5	10	4	4	8	8	8	8	

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6	$D_{5,4}$	2	1	
2	1 + 2 + 3 + 4	10	4	2	
3	1 + 3	5	8	3	G'
4	1 + 2	2	$M_{5,4}^{(3)}$	4	$Z = \Phi = \mathfrak{Z}$

Sylow subgroups: $[8] \times 5$, $[5]$ Maximal subgroups: $[D_{5,4}]$, $[8] \times 5$



$$40.12 = \langle A^2, B^5, C^2, D^2, DB = B^{-1}D, DC = ACD \rangle$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	A	B	AB	B ²	AB ²	C	BC	B ⁴ C	B ² C	B ³ C	A ^m B ⁿ D	A ^m B ⁿ CD	
		B ⁴	AB ⁴	B ³	AB ³	AC	AB ⁴ C	ABC	AB ³ C	AB ³ C			

C	1	2	3	4	5	6	7	8	9	10	11	12	13	
#	1	1	2	2	2	2	2	2	2	2	2	10	10	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	-1	-1	1
χ_3	1	1	1	1	1	1	-1	-1	-1	-1	-1	1	-1	2
χ_4	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	3
χ_5	2	2	c ₁	c ₁	c ₂	c ₂	2	c ₁	c ₁	c ₂	c ₂	0	0	4
χ_6	2	2	c ₁	c ₁	c ₂	c ₂	-2	c ₁	c ₁	-c ₂	-c ₂	0	0	5
χ_7	2	2	c ₂	c ₂	c ₁	c ₁	2	c ₂	c ₂	c ₁	c ₁	0	0	4
χ_8	2	2	c ₂	c ₂	c ₁	c ₁	-2	-c ₂	-c ₂	c ₁	c ₁	0	0	5
χ_9	2	-2	2	-2	2	-2	0	0	0	0	0	0	0	6
χ_{10}	2	-2	c ₁	-c ₁	c ₂	-c ₂	0	is ₁	-is ₁	is ₂	-is ₂	0	0	0
χ_{11}	2	-2	c ₁	-c ₁	c ₂	-c ₂	0	-is ₁	is ₁	-is ₂	is ₂	0	0	0
χ_{12}	2	-2	c ₂	-c ₂	c ₁	-c ₁	0	is ₂	-is ₂	-is ₁	is ₁	0	0	0
χ_{13}	2	-2	c ₂	-c ₂	c ₁	-c ₁	0	-is ₂	is ₂	is ₁	-is ₁	0	0	0
order	1	2	5	10	5	10	2	10	10	10	10	2	4	

$$c_k = 2\cos(2k\pi/5), s_k = 2\sin(2k\pi/5)$$

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11	10×2	2	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 12	$D_{10} \times 2$	2	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13	$D_{5,4}$	2	3	
4	1 + 2 + 7	2^2	10.2	4	
5	1 + 2	2	$D_{10} \times 2$	5	$Z = \Phi = \mathfrak{K}$
6	1 + 3 + 5	5	8.4	6	G'
7	1 + 2 + 3 + 4 + 5 + 6	10	2^2	1 \cap 2	

Sylow subgroups: $[D_8] \times 5$, $[5]$ Maximal subgroups: $[10 \times 2]$, $[D_{10} \times 2]$, $[D_{5,4}]$, $[D_8] \times 5$

$$40.13 = D_{40} = \langle A^{20}, B^2, BA = A^{-1}B \rangle$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	A^{10}	A^2	A^4	A^6	A^8	A	A^3	A^5	A^7	A^9	$A^{2n}B$	$A^{2n+1}B$	
		A^{18}	A^{16}	A^{14}	A^{12}	A^{19}	A^{17}	A^{15}	A^{13}	A^{11}			

C	1	2	3	4	5	6	7	8	9	10	11	12	13	
#	1	1	2	2	2	2	2	2	2	2	2	10	10	\mathfrak{X}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	-1	-1	1
χ_3	1	1	1	1	1	1	-1	-1	-1	-1	-1	1	-1	2
χ_4	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	3
χ_5	2	2	c_2	c_4	c_6	c_8	c_1	c_3	0	c_7	c_9	0	0	4
χ_6	2	2	c_4	c_8	c_8	c_4	c_2	c_6	-2	c_6	c_2	0	0	4
χ_7	2	2	c_6	c_8	c_2	c_4	c_3	c_9	0	c_1	c_7	0	0	4
χ_8	2	2	c_8	c_4	c_4	c_8	c_4	c_8	2	c_8	c_4	0	0	4
χ_9	2	-2	-2	2	-2	2	0	0	0	0	0	0	0	5
χ_{10}	2	-2	c_8	c_4	c_4	c_8	c_6	c_2	-2	c_2	c_6	0	0	0
χ_{11}	2	-2	c_6	c_8	c_2	c_4	c_7	c_1	0	c_9	c_3	0	0	0
χ_{12}	2	-2	c_4	c_8	c_8	c_4	c_8	c_4	-2	c_4	c_8	0	0	0
χ_{13}	2	-2	c_2	c_4	c_6	c_8	c_9	c_7	0	c_3	c_1	0	0	0
\wedge	1	2	10	5	10	10	20	20	4	20	20	2	2	

$$c_k = 2\cos(2k\pi/20)$$

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11	20	2	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 12	10 × 2	2	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13	10 × 2	2	3	
4	1 + 2 + 9	2	D₂₀	4	$Z = \Phi = \mathfrak{X}$
5	1 + 4 + 6	5	D₈	5	
6	1 + 2 + 3 + 4 + 5 + 6	10	2 × 2	1 ∩ 2	G'
7	1 + 2	4	D₁₀	3 ∩ 4	

Sylow subgroups: $[D_8] \times 5$, $[5]$ **Maximal subgroups:** $[20]$, $[D_{10} \times 2] \times 2$, $[D_8] \times 5$

$$40.14 = Q_{40} = \langle A^{20}, B^2 = A^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11	12	13
elts	1	A^{10}	A^2 A^{18}	A^4 A^{16}	A^6 A^{14}	A^8 A^{12}	A A^{19}	A^3 A^{17}	A^5 A^{15}	A^7 A^{13}	A^9 A^{11}	$A^{2n}B$	$A^{2n+1}B$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	
#	1	1	2	2	2	2	2	2	2	2	2	10	10	\mathfrak{X}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	-1	-1	1
χ_3	1	1	1	1	1	1	-1	-1	-1	-1	-1	1	-1	2
χ_4	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	3
χ_5	2	2	c_2	c_4	c_6	c_8	c_1	c_3	0	c_7	c_9	0	0	4
χ_6	2	2	c_4	c_8	c_8	c_4	c_2	c_6	-2	c_6	c_2	0	0	4
χ_7	2	2	c_6	c_8	c_2	c_4	c_3	c_9	0	c_1	c_7	0	0	4
χ_8	2	2	c_8	c_4	c_4	c_8	c_4	c_8	2	c_8	c_4	0	0	4
χ_9	2	-2	-2	2	-2	2	0	0	0	0	0	0	0	5
χ_{10}	2	-2	c_8	c_4	c_4	c_8	c_6	c_2	-2	c_2	c_6	0	0	0
χ_{11}	2	-2	c_6	c_8	c_2	c_4	c_7	c_1	0	c_9	c_3	0	0	0
χ_{12}	2	-2	c_4	c_8	c_8	c_4	c_8	c_4	-2	c_4	c_8	0	0	0
χ_{13}	2	-2	c_2	c_4	c_6	c_8	c_9	c_7	0	c_3	c_1	0	0	0
\wedge	1	2	10	5	10	10	20	20	4	20	20	4	4	

$$c_k = 2\cos(2k\pi/20)$$

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11	20	2	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 12	10×2	2	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13	10×2	2	3	
4	1 + 2 + 9	2	D_{20}	4	$Z = \Phi$
5	1 + 4 + 6	5	D_8	5	
6	1 + 2 + 3 + 4 + 5 + 6	10	2×2	$1 \cap 2$	G'
7	1 + 2	4	D_{10}	$4 \cap 5$	$Z_2 = \mathfrak{X}$

Sylow subgroups: $[Q_8] \times 5$, $[5]$ Maximal subgroups: $[20]$, $[D_{5,4}] \times 2$, $[Q_8] \times 5$

$$41 = \langle A^{41} \rangle$$

$$42 = \langle A^{42} \rangle = 21 \times 2$$

$$42.2 = D_{42} = \langle A^3, B^2, C^7, BA = A^{-1}B \rangle = D_6 \times 7$$

$$42.3 = D_{14} \times 3 = \langle A^7, B^2, C^3, BA = A^{-1}B \rangle$$

$$42.4 = M_{7,3}^{(4)} \times 2 = \langle A^7, B^3, C^2, BA = A^4B \rangle$$

$$42.5 = M_{7,6}^{(5)} = \langle A^7, B^6, BA = A^5B \rangle$$

CC	1	2	3	4	5	6	7
elts	1	A A ² A ³ A ⁴ A ⁵ A ⁶	A ⁿ B ²	A ⁿ B ⁴	A ⁿ B ³	A ⁿ B ⁵	A ⁿ B

C	1	2	3	4	5	6	7	
#	1	3	3	7	7	1	3	\mathcal{K}
χ_1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	-1	-1	-1	1
χ_3	1	1	ω	ω^2	1	ω	ω^2	2
χ_4	1	1	ω^2	ω	1	ω^2	ω	2
χ_5	1	1	ω	ω^2	-1	$-\omega$	$-\omega^2$	3
χ_6	1	1	ω^2	ω	-1	$-\omega^2$	$-\omega$	3
χ_7	6	-1	0	0	0	0	0	0
\wedge	1	7	3	3	2	6	6	

	Classes	H	G/H	\cap
1	1 + 2 + 3 + 4	$M_{7,3}^{(4)}$	2	1
2	1 + 2 + 5	D_{14}	3	2
3	1 + 2	7	6	3 G'

Sylow subgroups: $[2] \times 7$, $[3] \times 7$, $[7]$ Maximal subgroups: $[6] \times 7$, $[D_{14}]$, $[M_{7,3}^{(4)}]$



$$42.6 = D_{42} = \langle A^{21}, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11	12
elts	1	A ⁷ A ¹⁴	A ³ A ¹⁸	A ⁶ A ¹⁵	A ⁹ A ¹²	A A ²⁰	A ² A ¹⁹	A ⁴ A ¹⁷	A ⁵ A ¹⁶	A ⁸ A ¹³	A ¹⁰ A ¹¹	A ⁿ B

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	2	2	2	2	2	2	2	2	2	2	19	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	-1	1
χ_3	2	-1	2	2	2	-1	-1	-1	-1	-1	-1	0	2
χ_4	2	2	c ₃	c ₆	c ₉	c ₁	c ₂	c ₄	c ₅	c ₈	c ₁₀	0	3
χ_5	2	2	c ₆	c ₉	c ₃	c ₂	c ₄	c ₈	c ₁₀	c ₅	c ₁	0	3
χ_6	2	2	c ₉	c ₃	c ₆	c ₃	c ₆	c ₉	c ₆	c ₃	c ₉	0	3
χ_7	2	-1	c ₉	c ₃	c ₆	c ₄	c ₈	c ₅	c ₁	c ₁₀	c ₂	0	0
χ_8	2	-1	c ₆	c ₉	c ₃	c ₅	c ₁₀	c ₁	c ₃	c ₂	c ₈	0	0
χ_9	2	-1	c ₃	c ₆	c ₉	c ₆	c ₉	c ₂	c ₉	c ₆	c ₃	0	0
χ_{10}	2	-1	c ₃	c ₆	c ₉	c ₈	c ₅	c ₁₀	c ₂	c ₁	c ₄	0	0
χ_{11}	2	-1	c ₆	c ₉	c ₃	c ₉	c ₃	c ₆	c ₃	c ₉	c ₆	0	0
χ_{12}	2	-1	c ₉	c ₃	c ₆	c ₁₀	c ₁	c ₂	c ₈	c ₃	c ₅	0	0
\wedge	1	3	7	7	7	21	21	21	21	21	21	2	

$$c_k = 2\cos(2k\pi/21)$$

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11	21	2	1	G'
2	1 + 3 + 4 + 5	7	D ₆	2	
3	1 + 2	3	D ₁₄	3	

Sylow subgroups: [2] × 21, [3], [7] Maximal subgroups: [21], [D₁₄] × 21, [D₆] × 21

$$43 = \langle A^{43} \rangle$$

$$44 = \langle A^{44} \rangle = 11 \times 4$$

$$44.2 = 22 \times 2 = \langle A^{22}, B^2 \rangle$$

$$44.3 = D_{44} = D_{22} \times 2 = \langle A^{11}, B^2, C^2, BA = A^{-1}B \rangle$$

$$44.4 = D_{11,4} = \langle A^{11}, B^4, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14
elts	1	B ²	A A ¹⁰	A ² A ⁹	A ³ A ⁸	A ⁴ A ⁷	A ⁵ A ⁶	AB ² A ¹⁰ B ²	A ² B ² A ⁹ B ²	A ³ B ² A ⁸ B ²	A ⁴ B ² A ⁷ B ²	A ⁵ B ² A ⁶ B ²	A ⁿ B	A ⁿ B ³

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	2	2	2	2	11	1	2	2	2	2	11	11	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	1
χ_3	1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	i	-i	2
χ_4	1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-i	i	2
χ_5	2	2	c ₁	c ₂	c ₃	c ₄	c ₅	c ₁	c ₂	c ₃	c ₄	c ₅	0	0	3
χ_6	2	2	c ₂	c ₄	c ₅	c ₃	c ₁	c ₂	c ₄	c ₅	c ₃	c ₁	0	0	3
χ_7	2	2	c ₃	c ₅	c ₂	c ₁	c ₄	c ₃	c ₅	c ₂	c ₁	c ₄	0	0	3
χ_8	2	2	c ₄	c ₃	c ₁	c ₅	c ₂	c ₄	c ₃	c ₁	c ₅	c ₂	0	0	3
χ_9	2	2	c ₅	c ₁	c ₄	c ₂	c ₃	c ₅	c ₁	c ₄	c ₂	c ₃	0	0	3
χ_{10}	2	-2	c ₁	c ₂	c ₃	c ₄	c ₅	-c ₁	-c ₂	-c ₃	-c ₄	-c ₅	0	0	0
χ_{11}	2	-2	c ₂	c ₄	c ₅	c ₃	c ₁	-c ₂	-c ₄	-c ₅	-c ₃	-c ₁	0	0	0
χ_{12}	2	-2	c ₃	c ₅	c ₂	c ₁	c ₄	-c ₃	-c ₅	-c ₂	-c ₁	-c ₄	0	0	0
χ_{13}	2	-2	c ₄	c ₃	c ₁	c ₅	c ₂	-c ₄	-c ₃	-c ₁	-c ₅	-c ₂	0	0	0
χ_{14}	2	-2	c ₅	c ₁	c ₄	c ₂	c ₃	-c ₅	-c ₁	-c ₄	-c ₂	-c ₃	0	0	0
\wedge	1	2	11	11	11	11	11	22	22	22	22	22	2	2	

$$c_k = 2\cos(2k\pi/5)$$

	Classes										H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12										22	2	1	
2	1 + 3 + 4 + 5 + 6 + 7										11	4	2	G'
3	1 + 2										2	D ₂₂	3	Z

Sylow subgroups: [4] × 11, [11] Maximal subgroups:

$$45 = \langle A^{45} \rangle$$

$$45.2 = 15 \times 3 = \langle A^{15}, B^3 \rangle$$

$$46 = \langle A^{46} \rangle = 23 \times 2$$

$$46.2 = D_{46} = \langle A^{23}, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	
elts	1	A	A ²	A ³	A ⁴	A ⁵	A ⁶	A ⁷	A ⁸	A ⁹	A ¹⁰	A ¹¹	A ¹²	A ¹³ B

C	1	2	3	4	5	6	7	8	9	10	11	12	13	
#	1	2	2	2	2	2	2	2	2	2	2	2	23	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	-1	1
χ_3	2	c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	c ₇	c ₈	c ₉	c ₁₀	c ₁₁	0	0
χ_4	2	c ₂	c ₄	c ₆	c ₈	c ₁₀	c ₁₁	c ₉	c ₇	c ₅	c ₃	c ₁	0	0
χ_5	2	c ₃	c ₆	c ₉	c ₁₁	c ₈	c ₅	c ₂	c ₁	c ₄	c ₇	c ₁₀	0	0
χ_6	2	c ₄	c ₈	c ₁₁	c ₇	c ₃	c ₁	c ₅	c ₉	c ₁₀	c ₆	c ₂	0	0
χ_7	2	c ₅	c ₁₀	c ₈	c ₃	c ₂	c ₇	c ₁₁	c ₆	c ₁	c ₄	c ₉	0	0
χ_8	2	c ₆	c ₁₁	c ₅	c ₁	c ₇	c ₁₀	c ₄	c ₂	c ₈	c ₉	c ₃	0	0
χ_9	2	c ₇	c ₉	c ₂	c ₅	c ₁₁	c ₄	c ₃	c ₁₀	c ₆	c ₁	c ₈	0	0
χ_{10}	2	c ₈	c ₇	c ₁	c ₉	c ₆	c ₂	c ₁₀	c ₅	c ₃	c ₁₁	c ₄	0	0
χ_{11}	2	c ₉	c ₅	c ₄	c ₁₀	c ₁	c ₈	c ₆	c ₃	c ₁₁	c ₂	c ₇	0	0
χ_{12}	2	c ₁₀	c ₃	c ₇	c ₆	c ₄	c ₉	c ₁	c ₁₁	c ₂	c ₈	c ₅	0	0
χ_{13}	2	c ₁₁	c ₁	c ₁₀	c ₂	c ₉	c ₃	c ₈	c ₄	c ₇	c ₅	c ₆	0	0
\wedge	1	11	11	11	11	11	11	11	11	11	11	11	2	

Normal subgroups

Classes	H	G/H	\cap
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12	23	2

1 G'

Sylow and maximal subgroups: $[2] \times 23$, $[23]$

$$47 = \langle A^{47} \rangle$$

